EARTH SCIENCES
SCIENTIFIC BACKGROUND ASSESSMENT

Exercise 1: Vertical structure of the lower troposphere

In this exercise we will describe the vertical thermal structure of the Earth atmosphere in its lower layers. The meaning of symbols and variables, as well as the value of some physical constant are listed in Tab.1 .

The first law of themodynamics can be written

$$c_v dT + P d\alpha = J \tag{1}$$

Where dT and $d\alpha$ are the variations of temperature and specific volume (the inverse of density), p is pressure and J represents the external energy given to the system.

Symbol	Meaning	Value	
R	Ideal gas constant	287	$J\ kg^{-1}\ K^{-1}$
c_p	Specific heat of dry air at constant pressure	1005	$J\ kg^{-1}\ K^{-1}$
c_v	Specific heat of dry air at constant volume	717	$J\ kg^{-1}\ K^{-1}$
ρ	Air density	1	$Kg\ m^{-3}$
g	gravity acceleration at the Earth surface	9.81	$m\ s^{-2}$

Table 1: meaning of symbols and value of some constants

question 1

We will make the hypothesis that the atmosphere is dry, adiabatic and hydrostatic,

1.1 what is the variation of pressure with height, in the case of isothermal atmosphere?

1.2 Find the adiabatic lapse rate, i.e. the vertical derivative of atmospheric temperature.

hint: use eq.1 and the ideal gas law: $p\alpha = RT$.

Remember that the hydrostatic approximation can be written:

$$\frac{dp}{dz} = -\rho g.$$

question 2

2.1 If the average temperature of Earth at sea level is 15°C, what is the average temperature at 2000 meters of altitude in the above hypotheses?

2.2 The estimate above is actually way colder than observed. Why? What is the element that is missing?

question 3

Show that in the dry, hydrostatic and adiabatic hypothesis temperature can be expressed as a function of pressure $T(p)$ with the following formula:

$$T(p) = T_0 \left(\frac{p}{p_0} \right)^{\frac{R}{c_p}} \tag{2}$$

Where T_0 is the temperature at a given reference pressure level p_0.

$\theta = T(p) \left(\frac{p_0}{p} \right)^{\frac{R}{c_p}}$*, obtained inverting (2), is called potential temperature; it is the temperature that a parcel of air would have if displaced adiabatically to the reference level* p_0*, tipically 1000 mb. Its profile in the vertical direction tells us if the atmosphere is statically stable or unstable to small perturations. When the atmosphere is unstable, convective motion appears.*

question 4

4.1 In Fig.1, lines of constant θ are plotted in blue on a temperature-pressure plan. The lines are numbered; which line correspond to $\theta = 273.15°K$?

4.2 The red line of Fig.1 shows the measurements of temperature taken by a balloon at different pressure levels ascending from the ground. At which pressure levels is the atmosphere unstable?

question 5

What are convective movements? In which regions of the Earth is convection more frequent, and in which seasons?

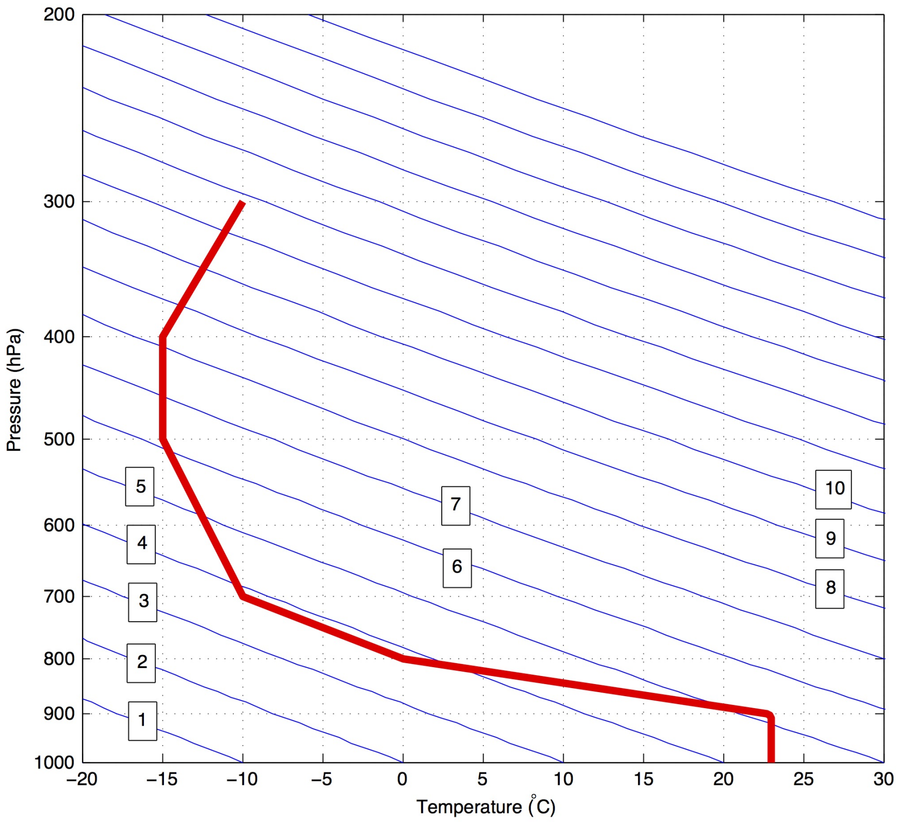

Figure 1: in blue: Temperature-Pressure plots of constant θ lines. In red: Temperature measures at different levels by a sounding balloon.

Exercise 2: Convection in the Earth's mantle

In 1931, Arthur Holmes is one of the rare advocates of Alfred Wegener's "Continental drift" theory. According to him, the Earth's mantle was heated by radioactive elements, and convection cells were bringing up the Earth's crust to surface.

The Rayleigh number can generally be expressed as:

$$Ra = \frac{\alpha_v (T_1 - T_o)\rho_o g L^3}{\mu \kappa} \qquad (1)$$

where α_v is the thermal expansion coefficient (K^{-1}), ρ_o the bulk density, g the standard gravity, $(T_1\text{-}T_o)$ the temperature gradient, μ the dynamic viscosity (Pa.s), κ the thermal diffusivity ($m^2.s^{-1}$) and L the characteristic length scale of the system considered.

Question 1

1.1 What is the physical unit and the meaning of the term $\alpha_v (T_1 - T_o)\rho_o g L$?

1.2 What is the physical meaning of $\mu\kappa / L^2$?

1.3 What does the Rayleigh number represent?

A system starts convecting for typical values of Ra >1000. When there is an internal source of heat, as for radioactive elements in rocky planet interiors, the Rayleigh number can be re-written in the following manner:

$$Ra_H = \frac{\alpha_v \rho_o^2 g H L^5}{\mu \kappa k} \qquad (2)$$

where H is the thermal power dissipated by unit mass ($W.kg^{-1}$) and k the thermal conductivity ($W.m^{-1}.K^{-1}$). Other parameters are the same as in equation (1).

The following consists in calculating – *independently* - an approximate value of each of the parameters above, in order to constrain the Rayleigh number in the Earth's mantle.

Question 2

We start first with the average bulk density. The moment of inertia *C* of a sphere of radius *a* and radial bulk density distribution $\rho(r)$ is given by:

$$C = \frac{8\pi}{3}\int_0^a \rho(r) r^4 dr \quad (3)$$

2.1 For a constant density ρo, calculate the normalized moment of intertia C/Ma^2 ?

2.2 The following table gives measured values of C/Ma^2 on Earth, Mars, Venus and the Moon. Comment.

	Radius (km)	Mass (kg)	C/Ma^2
Earth	6370	$5{,}97\text{x}10^{24}$	0,33
Lune	1730	$7{,}34\ \text{x}10^{22}$	0,39
Venus	6051	$4{,}86\text{x}10^{24}$	0,36
Mars	3390	$0{,}64\text{x}10^{24}$	0,33

2.3 Assuming the Earth's core (approximately 3400km in radius) has an average density of 10, calculate the mass and then the average bulk density of the Earth's mantle (the existence of a low density crust will be neglected in this calculation).

Question 3

We now aim at calculating the thermal power *H*. In order to do so, geophysicists measure the surface heat flow on Earth (figure 2).

3.1 Where are the areas of high surface heat flow on Earth?

3.2 Where are the areas of low surface heat flow on Earth?

3.3 We estimate the average surface heat flow on Earth equal to $65 mW/m^2$. What is the average thermal power *H* ($W.kg^{-1}$) dissipated by the Earth.

3.4 Approximately 20% of this heat is produced within the crust, while another 20% comes from the secular cooling of the planet. In addition, we assume that the core does not contain any radioactive elements. Calculate the average thermal power *H* dissipated within the mantle only.

Question 4

The Earth's mantle heat production is due to the slow radioactive decay of the Uranium isotopes ^{235}U and ^{238}U, and Thorium isotope ^{232}Th. The thermal power and radioactive half-life of these isotopes are given in the table below:

	H ($W.kg^{-1}$)	$T_{1/2}$ (years)
^{238}U	$9{,}46\text{x}10^{-5}$	$4{,}47\text{x}10^{9}$
^{235}U	$5{,}69\text{x}10^{-4}$	$7{,}04\text{x}10^{8}$
^{232}Th	$2{,}64\text{x}10^{-5}$	$1{,}4\text{x}10^{10}$

4.1 At present, natural uranium is composed of 99,28% ^{238}U and 0.71% ^{235}U, while natural thorium is composed solely of the single isotope ^{232}Th. In addition, geochemical studies performed on mantle rocks have shown that the present day concentrations of uranium and thorium are $C^{Th}_o/C^{U}_o \approx 4$. Calculate the concentrations of natural uranium and thorium in the Earth's mantle.

4.2 Recalling $C(t) = C_o \exp(-0.69\ t / T_{1/2})$, i.e. the relationship between the concentration of an isotope at a given time t, its present day concentration and its radio-active half-life, calculate the thermal power H of the Earth's mantle at any given time of its geological history.

4.3 When was the Earth formed and what was the thermal power of the Earth's mantle then? Comment.

Question 5

We now aim at calculating the viscosity of the Earth's mantle. To do so, geophysicists use models of viscous deformation, in which the vertical displacement of a rigid surface "floating" on a viscous body is shown to decrease exponentially as the viscous underlying body is flowing. The topography variation can thus be expressed according to the following formula:

$$h=h_m\exp(-t/\tau), \text{ où } \tau = 4\pi\mu/\rho g\lambda \qquad (4)$$

where μ is the dynamic viscosity, g the standard gravity, ρ the bulk density of the solid and λ the characteristic length of the rigid surface.

5.1 Figure 3a shows elevated paleo - (*old*) shorelines in northern Sweden. Why is there an uplift of 2 mm/year measured everywhere in Scandinavia nowadays?

5.2 On figure 3b, the dark curve displays the best fit obtained using the exponential model above. Determine the characteristic relaxation time τ.

5.3 In the case of the Scandinavia, a reasonable wavelength λ is of the order of 3000km. Calculate an order of magnitude for the Earth mantle's viscosity.

Question 6

The other parameters in equation (2) can all be measured experimentally. For olivine, the main mineral constituent of the Earth's mantle, the thermal conductivity is $k \approx 4\ W.m^{-1}.K^{-1}$, the thermal diffusivity $\kappa \approx 1\ mm^2.s^{-1}$ and the thermal expansion $\alpha_v \approx 3x10^{-5}\ K^{-1}$.

6.1 Using the above estimates for the other parameters, calculate the Rayleigh number for the Earth's mantle. Comment.

6.2 Using the results from question 4b, give an estimate of when will the mantle convection stop on Earth? Comment.

6.3 What are the many possible limitations of our calculation?

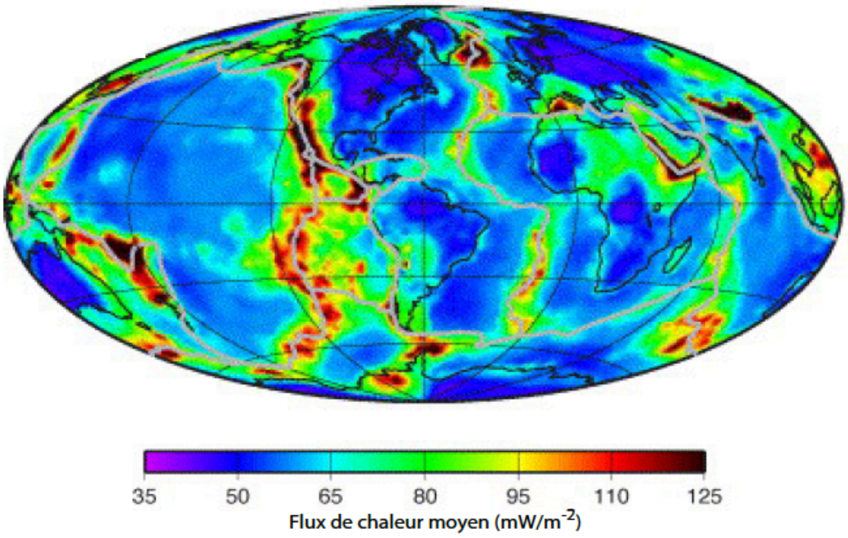

Figure 2. Average surface heat flux (mW/m^2) measured on Earth.

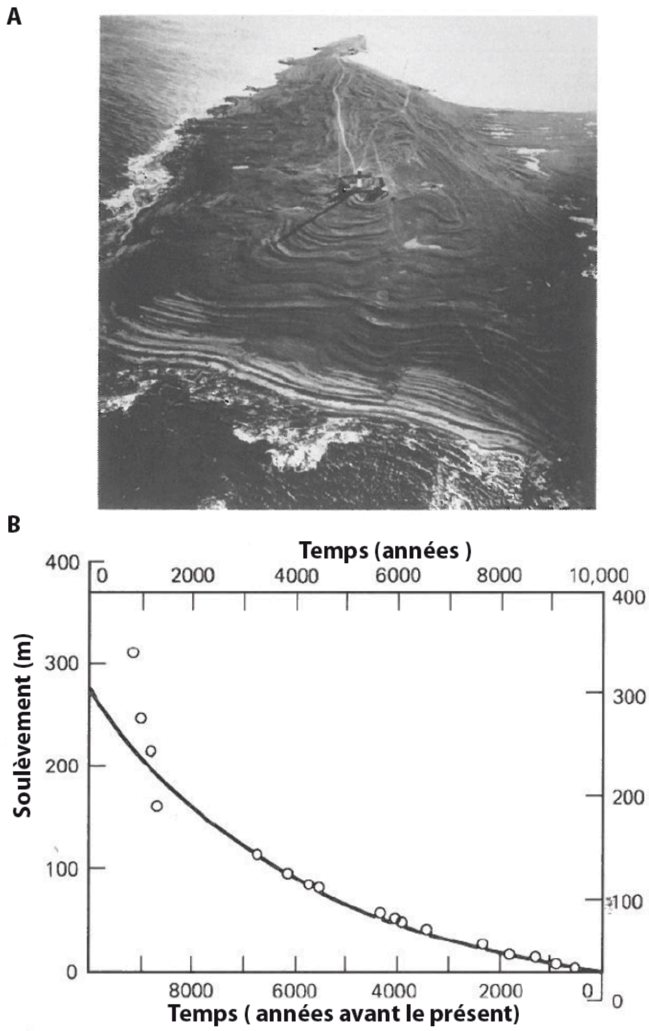

Figure 3. A) Paleo-shorelines in Östergransholm, Sweden. The vertical uplift there is 2mm/year. B) Total vertical uplift (in meters) measured at the mouth of the Angerman river, in Sweden, for the last 10 000 years. Dots indicate measures. The black curve is a best fit for an exponential decay.