# SOLVABLE AND UNSOLVABLE PROBLEMS

By Alan Mathison Turing

*The candidates must comment this text on the base of their personal philosophical and historical culture. Illustrations drawn from their proper (and similar) discipline are welcome.*

“If one is given a puzzle to solve one will usually, if it proves to be difficult, ask the owner whether it can be done. Such a question should have a quite definite answer, yes or no, at any rate provided the rules describing what you are allowed to do are perfectly clear. Of course the owner of the puzzle may not know the answer. One might equally ask, ‘How can one tell whether a puzzle is solvable?’, but this cannot be answered so straightforwardly. The fact of the matter is that there is *no* systematic method of testing puzzles to see whether they are solvable or not. If by this one meant merely that nobody had ever yet found a test which could be applied to any puzzle, there would be nothing at all remarkable in this statement. It would have been a great achievement to have invented such a test, so we can hardly be surprised that it has never be done. But it is not merely that the test has never been found. It has been proved that no such test ever can be found.

[…] Puzzles where one is asked to separate rigid bodies are in a way like the ‘puzzle’ of trying to undo a tangle, or more generally of trying to turn one knot into another without cutting the string. The difference is that one is allowed to bend the string, but not the wire forming the rigid bodies. In either case, if one wants to treat the problem seriously and systematically one has to replace the physical puzzle by a mathematical equivalent. The knot puzzle lends itself quite conveniently to this. A knot is just a closed curve in three dimensions nowhere crossing itself; but, for the purpose we are interested in, any knot can be given accurately enough as a series of segments in the directions of the three coordinate axes. Thus, for instance, the trefoil knot (Figure 1*a*) may be regarded as consisting of a number of segments joining the points given, in the usual ($x$, $y$, $z$) system of coordinates, as (1, 1, 1), (4, 1, 1), (4, 2, 1), (4, 2, – 1), (2, 2, – 1), (2, 2, 2), (2, 0, 2), (3, 0, 2), (3, 0, 0), (3, 3, 0), (1, 3, 0), (1, 3, 1) and returning again with a twelfth segment to the starting point (1, 1, 1). This representation of the knot is shown in perspective in Figure 1*b*.

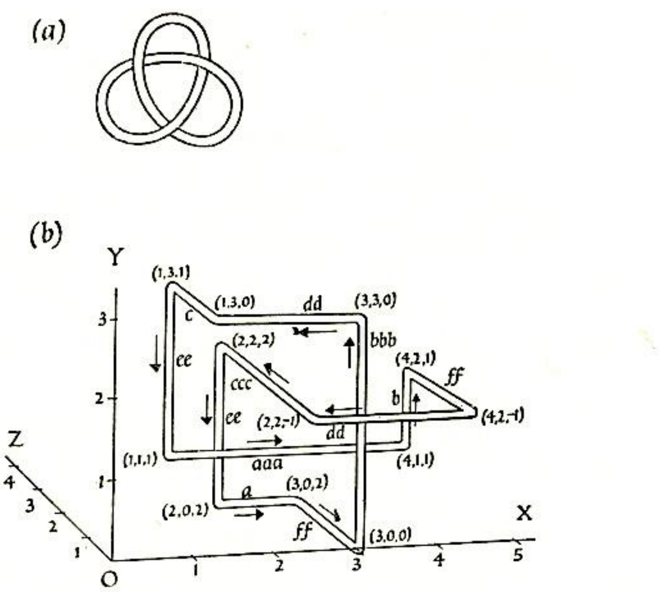


Fig. 1. (*a*) The trefoil knot (*b*) a possible representation of this knot as a number of segments joining points.

[…] First of all we may suppose that the puzzle is somehow reduced to a mathematical form in the sort of way that was used in the case of the knots. The position of the puzzle may be described, as was done in that case, by sequences of symbols in a row. There is usually very little difficulty in reducing other arrangements of symbols to this form. The question which remains to be answered is 'What sort of rules should one be allowed to have for rearranging the symbols or counters?' In order to answer this one needs to think about what kinds of processes ever do occur in such rules, and, in order to reduce their number, to break them up into simpler processes. Typical of such processes are counting, copying, comparing, substituting. When one is doing such processes, it is necessary, especially if there are many symbols involved, and if one wishes to avoid carrying too much information in one's head, either to make a number of jottings elsewhere or to use a number of marker objects as well as the pieces of the puzzle itself. For instance, if one were making a copy of a row of counters concerned in the puzzle it would be as well to have a marker which divided the pieces which have been copied from those which have not and another showing the end of the portion to be copied. Now there is no reason why the rules of the puzzle itself should not be expressed in such a way as to take account of these markers. If one does express the rules in this way they can be made to be just substitutions. This means to say that the *normal form for puzzles is the substitution type of puzzle*.

[…] It is clear that the difficulty in finding decision procedures for types of puzzle [it should notice that a decision problem only arises when one has an infinity of questions to ask] lies in establishing that the puzzle is unsolvable in those cases where it *is* unsolvable. This requires some sort of mathematical argument. This suggests that we might try expressing the statement that the puzzle comes out in a mathematical form and then try and prove it by some systematic process. There is no particular difficulty in the first part of this project, the mathematical expression of the statement about the puzzle. But the second half of the project is bound to fail, because by a famous theorem of Gödel no systematic method of proving mathematical theorems is sufficiently complete to settle every mathematical question, yes or no".

Alan Mathison TURING, *Solvable and Unsolvable Problems*, Science News 31, 1954, pp. 1, 3-4.

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Alan Mathison TURING, (23 June, 1912 – 7 June, 1954) was a British mathematician, logician, cryptanalyst and computer scientist. Turing is often considered to be the father of modern *computer science*. He provided an influential formalisation of the concept of the *algorithm* and *computation* with the *Turing machine*. Of his role in the modern computer, Time Magazine in naming Turing one of the 100 most influential people of the 20th century, states: "The fact remains that everyone who taps at a keyboard, opening a spreadsheet or a word-processing program, is working on an incarnation of a Turing machine." With the *Turing test*, meanwhile, he made a significant and characteristically provocative contribution to the debate regarding artificial intelligence: whether it will ever be possible to say that a machine is conscious and can think. During the Second World War, Turing was for a time head of Hut 8, the section responsible for German naval cryptanalysis. He devised a number of techniques for breaking German ciphers, including the method of the bombe, an electromechanical machine that could find settings for the *Enigma* machine.

Near the end of his life Turing became interested in chemistry. He wrote a paper on the chemical basis of *morphogenesis* and he predicted oscillating chemical reactions such as the Belousov–Zhabotinsky reaction, which were first observed in the 1960s.