International Selection: Physics (Minors)

Exercise 1: Ideal Gas

We consider two containers of volumes V_1 and V_2 respectively. The first one contains carbon dioxide, under pressure p_1. The second one contains oxygen molecules, under pressure p_2.

1. In this exercise, the gases will be considered as ideal gases. Explain this statement.
2. We consider the whole system at the temperature $t = 0^o$ C. We connect the two containers by a very thin tube. At equilibrium, still at the same temperature of $t = 0^o$ C, calculate the partial pressures p_{p1} of carbon dioxide and p_{p2} of oxygene in the mixture.
3. What is the total pressure of the mixture? What is the total mass of the mixture?
4. We heat the mixture to a temperature of $t = 15^o$ C. We neglect the thermal expansion of the containers. Give the total pressure of the mixture and its total mass.
5. Reconsider the above questions, using the following values: $V_1 = 3$ liters, V_2= 1 liter, p_1= 4 atm, p_2= 6 atm, M_{CO_2}=44g, $\mathrm{M}_{\mathrm{O}_2}$=32g,

Exercise 2: Real gases

The Mariotte temperature is defined as the temperature at which the behavior of a real gas is closest to the one of an ideal gas.

1. Find the Mariotte temperature for the following gases:
   - a gas described by the equation of state
     $$(p + \frac{n^2 a}{V^2})(V - nb) = nRT \tag{1}$$
   - a gas described by the equation of state
     $$(p + \frac{n^2 a'}{TV^2})(V - nb) = nRT \tag{2}$$
   - a gas described by the equation of state
     $$p(V - nb)exp(\frac{na}{RTV}) = nRT. \tag{3}$$

Exercise 3: Euler-Lagrange Equations

We consider a point mass (mass m) described by its position coordinate x, and the time derivative $\dot{x}$ of this coordinate. The mass is placed in a potential $V(x)$. Consider the function $L = \frac{m}{2}\dot{x}^2 - V(x)$.

1. Write out the "Euler-Lagrange equation":

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0 \tag{4}$$

   Give a physical interpretation of the resulting equation.

2. What happens if V does not depend on x ?

Exercise 4: Top

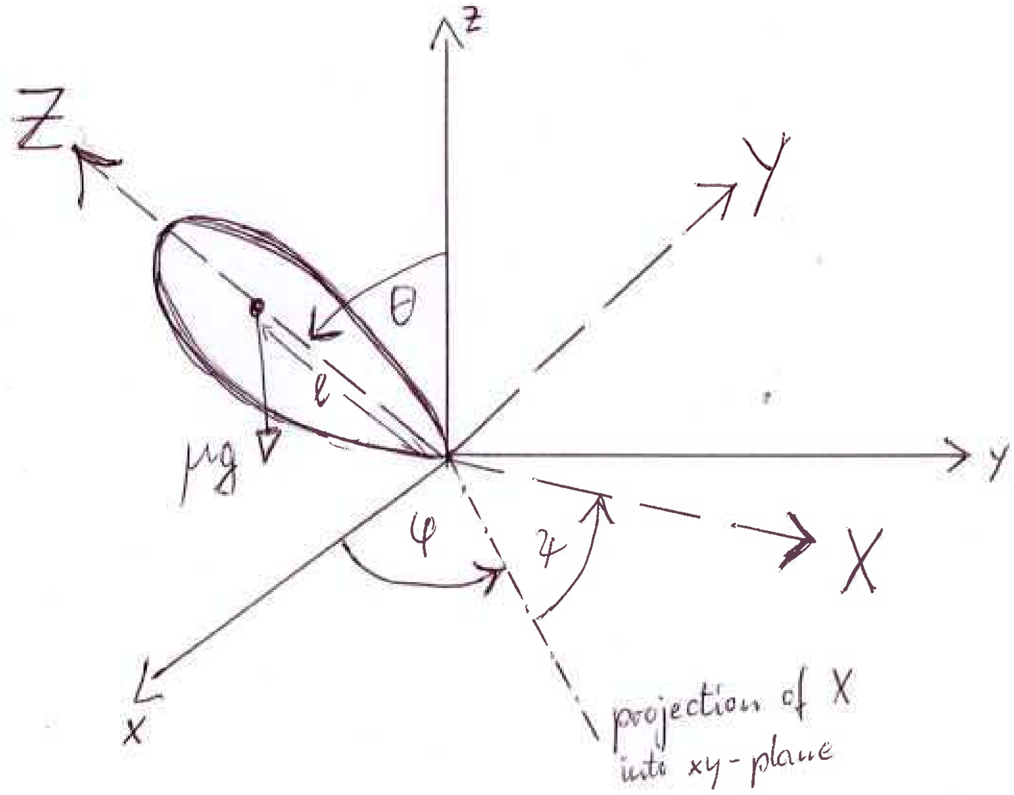
We consider a top (or gyroscope) of mass μ in the gravity field of the earth (we denote the gravity acceleration as g). The top is rotationally invariant around one of its axes, which means that it has two moments of inertia that are equal and the center of gravity lies on the symmetry axis. The lowest point of the top is immobile.

It is convenient to choose this point as the common origin of two Cartesian coordinate systems, one fixed in space and one moving with the top. The position of the top is described by three Euler angles, see figure. The motion of the top can be described by the function

$$L = \frac{I_1 + \mu l^2}{2}(\dot{\theta}^2 + \dot{\phi}^2 sin^2(\theta)) + \frac{I_3}{2}(\dot{\psi} + \dot{\phi}cos(\theta))^2 - \mu glcos(\theta) \tag{5}$$

where l is the distance of the centre of mass from the origin.

1. Explain the preceding statement.
2. Can you identify two conserved quantities? (Hint: Euler-Lagrange equations!)
3. Give a physical interpretation of these two quantities.
4. Derive the equation of motion for the remaining degree of freedom. Show that this equation can be obtained from an effective one-dimensional potential for the angle θ.
5. Describe the resulting motion (in words, no equations!).
6. Under which condition is a motion around a vertical axis ($\theta = 0$) stable?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